Creep motion in a random-field Ising model

L. Roters,* S. Lübeck,† and K. D. Usadel‡
Theoretische Tieftemperaturphysik, Gerhard-Mercator-Universität Duisburg, 47048 Duisburg, Germany
(Received 23 June 2000; published 24 January 2001)

We analyze numerically a moving interface in the random-field Ising model which is driven by a magnetic field. Without thermal fluctuations the system displays a depinning phase transition, i.e., the interface is pinned below a certain critical value of the driving field. For finite temperatures the interface moves even for driving fields below the critical value. In this so-called creep regime the dependence of the interface velocity on the temperature is expected to obey an Arrhenius law. We investigate the details of this Arrhenius behavior in two and three dimensions and compare our results with predictions obtained from renormalization group approaches.

DOI: 10.1103/PhysRevE.63.026113 PACS number(s): 64.60.Ht, 68.35.Rh, 68.35.Ja, 75.10.Hk

I. INTRODUCTION

In recent years the understanding of driven interfaces has improved considerably. Well known models of such interfaces are the equations of Edwards and Wilkinson [1] as well as of Kardar, Parisi, and Zhang [2]. Of particular interest are driven interfaces moving through a quenched disordered medium, which exhibit a so-called depinning phase transition. Without disorder the velocity of a driven interface grows linearly with the applied driving force or driving field, respectively. This behavior changes in the presence of quenched disorder. For small driving fields the interface is pinned by the disorder. The interface moves only if the driving field exceeds a critical value, i.e., on increasing the driving field a continuous phase transition from a pinned to a moving interface takes place (see, for instance, [3] and references therein). The expected dependence of the interface velocity on the driving field is sketched in Fig. 1. For very large driving fields the disorder can be neglected and consequently the velocity depends linearly on the driving field. The depinning transition happens due to the competition between the disorder and the driving field. The disorder induces some effective energy barriers that suppress the interface motion. The driving field reduces these energy barriers but they are overcome only if the driving field exceeds the critical value. Examples of systems exhibiting a depinning transition are charge density waves [4,5] or field driven domain walls in ferromagnets [6].

However, in the above scenarios thermal fluctuations are neglected. In real systems these fluctuations occur and no critical behavior is observed for finite temperatures. The reason is that even below the critical driving field the energy barriers can be overcome due to thermal fluctuations, resulting in a moving interface. A striking effect of thermal fluctuations occurs at the critical field where the interface velocity v depends on the temperature T according to $v \sim T^{1/\delta}$ with an exponent $\delta \geqslant 1$ [5,7–9]. Another effect of thermal fluctuations is the so-called creep motion that occurs for driving fields H well below the critical threshold at sufficiently low temperatures. Here, the interface velocity is expected to be characterized by an Arrhenius behavior

$$v \sim e^{-E(H)/T} \tag{1}$$

with a certain field dependent energy barrier $E(H)$. Creep motion was investigated, for instance, within the theory of flux creep phenomena [10] and in several renormalization group approaches to the Edwards-Wilkinson equation [11,12]. Experimentally, the Arrhenius behavior of the creep regime was observed for magnetic domain wall motion in thin films composed of Co and Pt layers [13].

In this paper we consider the interface motion occurring in a driven random-field Ising model (RFIM) in the creep regime. In the next section we describe the details of the model and the simulations. In Secs. III and IV we investigate numerically the creep motion of the interface in the two and three dimensional RFIM. In particular, we show that the velocity behavior can be described by an Arrhenius *Ansatz* and we investigate the temperature and field dependence of the

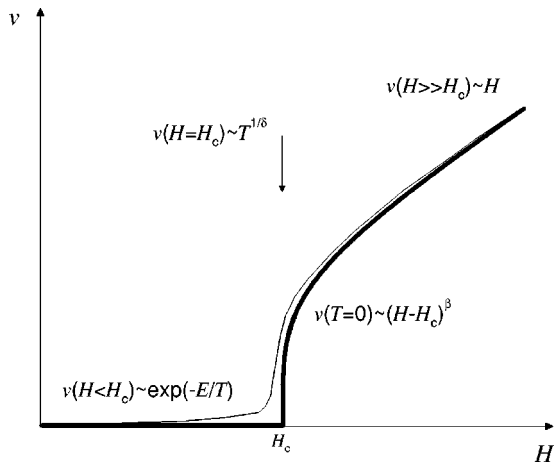

FIG. 1. Schematic sketch of the interface velocity v of the pinning phase transition and its dependence on the driving field H. The bold line corresponds to zero temperature, $T=0$. For small but finite temperatures the critical behavior is smeared out (thin solid line). The creep regime for small driving fields is characterized by an Arrhenius like behavior with an effective energy barrier E.

*Email address: lars@thp.uni-duisburg.de
†Email address: sven@thp.uni-duisburg.de
‡Email address: usadel@thp.uni-duisburg.de

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prefactor of the Arrhenius law and study the energy barrier $E(H)$. In Sec. V we summarize and discuss our results.

II. MODEL AND SIMULATIONS

To study the creep regime we consider the RFIM on a square or simple cubic lattice of linear size L. The Hamiltonian of the RFIM is given by

$$\mathcal{H}=-\frac{J}{2}\sum_{\langle i,j\rangle} S_iS_j-H\sum_i S_i-\sum_i h_iS_i, \tag{2}$$

where the first term characterizes the exchange interaction of neighboring spins ($S_i=\pm 1$). The sum is taken over all pairs of nearest neighbor spins. The spins are coupled to a homogeneous driving field H and to quenched random fields h_i which we choose to be uncorrelated ($\langle h_ih_j\rangle \propto \delta_{ij}$) with $\langle h_i\rangle = 0$. Throughout this paper we consider uniformly distributed disorder, i.e., the probability density p that the random field takes some value h_i is given by

$$p(h_i)=\begin{cases}(2\Delta)^{-1} & \text{for } |h_i|<\Delta\\ 0 & \text{otherwise.}\end{cases} \tag{3}$$

Using antiperiodic boundary conditions an interface is induced in the system which can be driven by the field H (see [9] for details). Starting with an initially flat interface we apply a Glauber dynamics with random sequential update and heat-bath transition probabilities (see, for instance, [14]). In our simulations the interface moves along the [11] and [111] direction of a simple cubic lattice. This is a natural choice since in the absence of disorder interface motion occurs for any finite driving field [8]. This property is an advantage, especially in the creep regime where the interface is driven by small driving fields at low temperatures.

The basic quantity in our simulations is the velocity of the moving interface, which is determined in the following way. The interface movement corresponds to an increasing magnetization which is monitored as a function of time, i.e., the number of Monte Carlo steps per spin. Starting from a flat interface the system after a certain transient regime reaches a steady state where the average magnetization grows linearly in time. The velocity of the interface is defined as the time derivative of this magnetization. Spin flips outside the interface may also occur, caused by the finite temperature. These isolated, rare spin flips are unstable for sufficiently small temperatures, i.e., they flip back in the next update. Thus these spin flips do not affect the measurement of the global average magnetization time dependence and therefore do not affect the determination of the interface velocity.

During its motion the originally flat interface roughens due to the disorder. The width of the interface increases and finally reaches a stationary state. For the data presented in this paper we have verified that the interface width remains small as compared to the extension of the system perpendicular to the interface.

III. CREEP MOTION IN THE TWO DIMENSIONAL RFIM MODEL

We measured the velocity of the interface in the creep motion regime. Since the creep regime is ''far away'' from the critical point, we expect that finite-size effects can be neglected. Investigations of various system sizes confirm this assumption and we use therefore in our simulations a large number of update steps instead of large system sizes. We performed for each temperature and field value at least 10^5 Monte Carlo steps. Additionally we focus our analysis on one value of the disorder strength ($\Delta=1.2$). We have also performed some spot checks at different values of Δ in order to confirm that the results are not sensitive to the disorder strength.

As mentioned above the velocity is expected to obey an Arrhenius law

$$v(H,T)=C(H,T)e^{-E(H)/T} \tag{4}$$

in the creep regime. The effective energy barrier $E(H)$ is independent of the temperature and tends to zero for $H\rightarrow H_c$. Following a renormalization group analysis [11] we assume that the temperature dependence of the prefactor of the Arrhenius law is characterized by a power-law behavior

$$C(H,T)=c(H)T^{-x} \tag{5}$$

with some particular exponent x. Independent of the actual value of x the interface motion stops for any finite value of the energy barrier ($H<H_c$) in the limit $T\rightarrow 0$.

In the first step of our analysis we determine the exponent x. In an Arrhenius plot $\ln vT^x$ vs $1/T$ the exponent x is varied until straight lines are obtained. Good results are found for $x=0.89\pm 0.17$ and the corresponding curves are shown in Fig. 2.

A regression analysis of these curves then yields the value of the prefactor $c(H)$ and the value of the energy barrier $E(H)$ [Eqs. (4) and (5)]. The results are plotted in Fig. 3. On increasing the driving field the effective energy barrier decreases as expected. But the prefactor of the Arrhenius law displays no significant field dependence, i.e., $c(H)=\text{const}$. This is confirmed by Fig. 4 which was obtained by plotting $\ln vT^x$ as a function of $E(H)/T$. The curves for different values of the driving field are seen to coalesce to a single curve, which happens only if $c(H)=\text{const}$.

We analyze the field dependence of the energy barrier starting from a recently performed renormalization group approach [11], assuming

$$E(H)=E_0\left[\left(\frac{H_c}{H}\right)^{\mu}-1\right]. \tag{6}$$

On approaching the critical field of the depinning transition H_c the energy barrier vanishes. The value of the critical field $H_c=1.12\pm 0.02$ is obtained from an independent simulation at zero temperature. Thus we plotted the rescaled velocities as a function of $(H_c/H)^{\mu}-1$ and tried to obtain a coalescence of the data similar to Fig. 4 by varying the exponent μ.

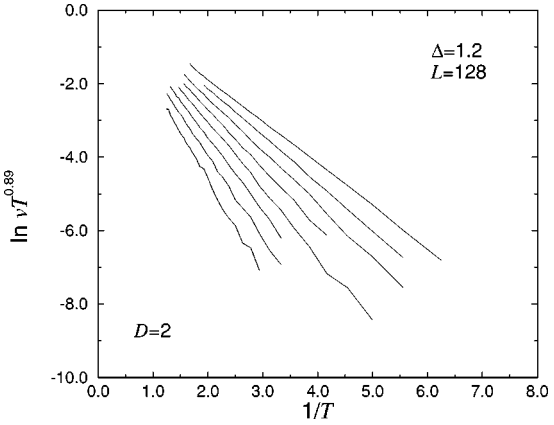

FIG. 2. The interface velocity v as a function of the temperature T for various values of the driving field ($H \in \{0.25,0.3,0.35,\ldots,0.6\}$ from bottom to top) and $\Delta=1.2$. According to Eq. (4) we plot $\ln vT^x$ vs $1/T$. On varying the exponent x we obtained nearly straight lines for $x=0.89\pm0.17$. The cutoffs at low and high temperatures are caused by different effects. At low temperatures the interface can be pinned for finite time intervals depending on the particular disorder configuration. In this case the interface displays a stop and go behavior which results in strong velocity fluctuations (not shown for clarity). The cutoff for large v occurs because the creep regime is exited at high temperatures or high driving fields.

(See Fig. 5). Our analysis shows that only a logarithmic field dependence for $\mu\to0$ fits the data, i.e.,

$$E(H)=E_0\ln\left(\frac{H_c}{H}\right) \tag{7}$$

(see Fig. 6). As one can see in Fig. 6 the logarithmic *Ansatz* yields a quite convincing fit. Our analysis therefore suggests that the effective energy barrier displays a logarithmic field dependence.

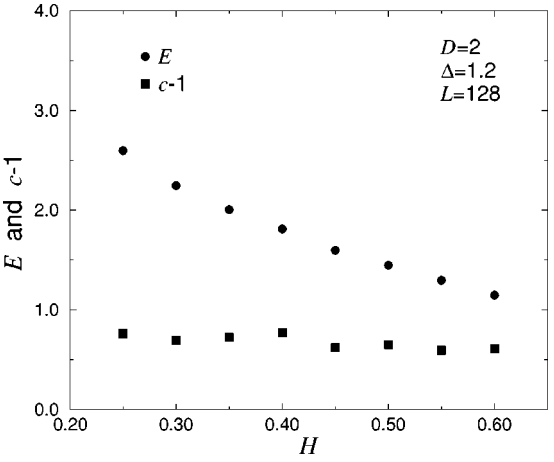

FIG. 3. The energy barrier E and the prefactor c [see Eqs. (4) and (5)] as a function of the driving field H. To avoid an overlap between the two curves we plot $c-1$ instead of c.

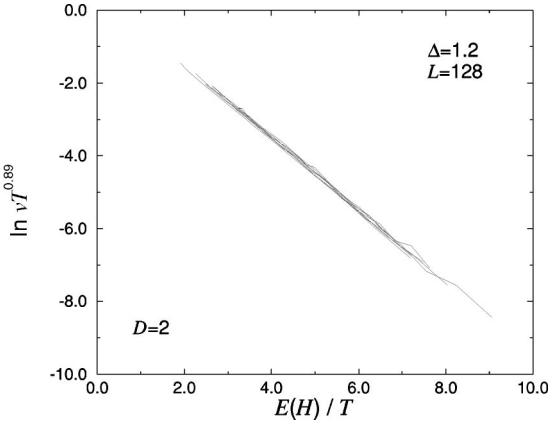

FIG. 4. The rescaled interface velocity $\ln vT^x$ vs $E(H)/T$. The values of the energy barrier $E(H)$ are obtained from a regression analysis of the corresponding curves of Fig. 2. The data collapse confirms that the prefactor c displays no significant field dependence [Eqs. (4) and (5)].

IV. THE THREE DIMENSIONAL MODEL

We next analyze the interface velocity of the three dimensional model for $\Delta=1.7$. For this value of the disorder the critical behavior has been investigated and the corresponding critical field has been found to be $H_c=1.37\pm0.01$ [9]. As for the two dimensional model we have also performed some simulations for values different from $\Delta=1.7$ to ensure that the main results discussed below do not depend on the particular choice of Δ.

By driving the interface at finite temperatures and fields below the critical threshold we measured the velocity $v(H,T)$. Again, we fitted the simulation data to Eqs. (4) and (5) by varying x to get straight lines in the $\ln vT^x$ vs $1/T$ plot. A good fit is obtained using $x=0.79\pm0.09$ (Fig. 7).

The result of the regression analysis for $E(H)$ and $c(H)$ is shown in Fig. 8. As in the two dimensional case $E(H)$ decreases with increasing driving field and the prefactor is

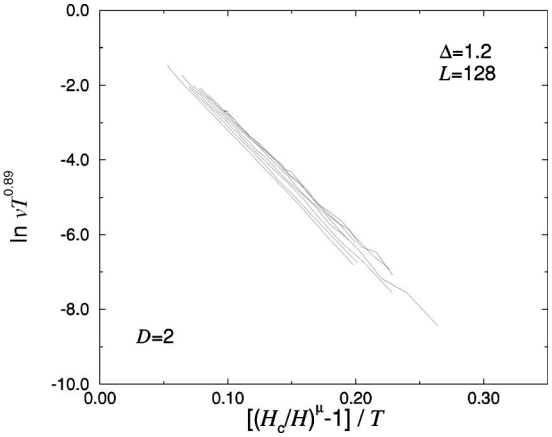

FIG. 5. The rescaled interface velocity $\ln vT^x$ as a function of $[(H_c/H)^\mu-1]/T$ [see Eq. (6)]. No coalescence of the data could be obtained for any finite value of μ. But with decreasing exponent μ the coalescence becomes better. The figure shows the corresponding curves for $\mu=0.05$.

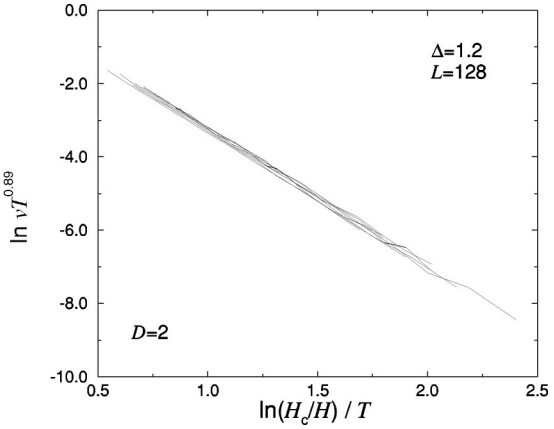

FIG. 6. The rescaled interface velocity $\ln vT^{x}$ as a function of $\ln(H_c/H)/T$. This logarithmic *Ansatz* yields a quite convincing data collapse, i.e., this result suggests that the field dependence of the energy barrier is given by Eq. (6) in the limit $\mu \to 0$.

essentially independent of the driving field. On plotting the rescaled interface velocities vs $E(H)/T$ the data coalesce (see Fig. 9) only if $c(H) = \text{const}$.

We consider now the field dependence of the energy barrier. In analogy to the previous section we check the conjectured field dependence obtained from a renormalization group approach. Applying the data of the interface velocities to the ansatz Eq. (6) yields a similar result as in two dimensions, i.e., the accuracy of the data collapse increases for $\mu \to 0$. Therefore we again assume that the dependence of the energy barrier on the driving field displays a logarithmic behavior [Eq. (7)]. The corresponding curves are shown in Fig. 10. As can be seen the logarithmic *Ansatz* yields a good fit of the velocity data in the creep regime.

But we have to admit that in contrast to the two dimensional case we observe here that different *Ansätze* for the field dependence of the energy barrier may also lead to a fit of the data. For example, several authors conjectured that the energy barrier is given by

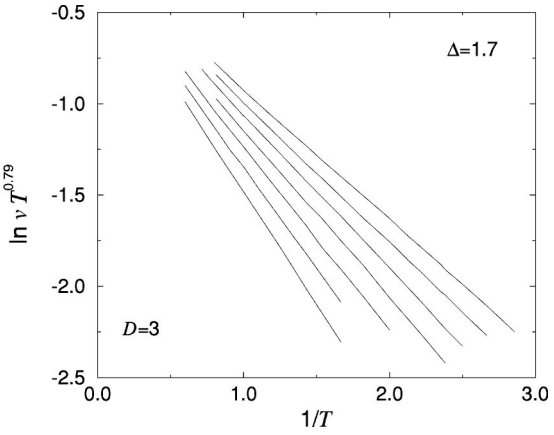

FIG. 7. Interface velocities of the three dimensional model for different temperatures and driving fields ($H = 0.3, 0.35, 0.4, \ldots, 0.6$, from bottom to top). On varying the exponent x we obtained nearly straight lines for $x = 0.79 \pm 0.09$.

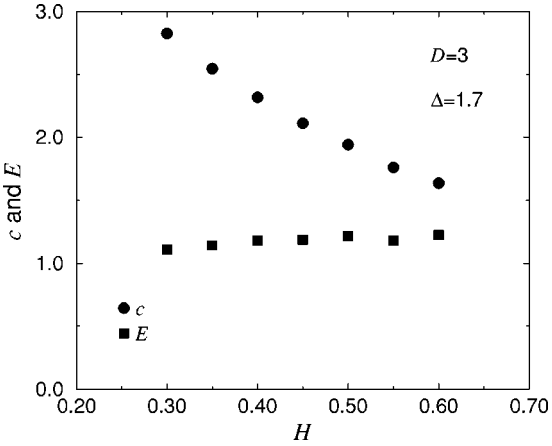

FIG. 8. The energy barrier E and the prefactor c [see Eq. (5)] as a function of the driving field H.

$$E(H) = E_0 \left(\frac{H_c}{H} \right)^{\mu} \tag{8}$$

for $H \ll H_c$. Note that Eq. (8) agrees with the one discussed above [Eq. (6)] for sufficiently small driving fields. The above *Ansatz* was derived within a theory of flux creep behavior [10] and is expected to hold for the present situation of driven interfaces. In this case the exponent μ is given by $\mu = (2\zeta + D - 3)/(2 - \zeta)$ with ζ denoting the roughness exponent of the interface at the depinning transition (see [12,13] and references therein). For the Edwards-Wilkinson equation with quenched disorder, $\zeta = (5 - D)/3$ has been determined by an ϵ expansion within a renormalization group scheme [15]. This value is believed to be exact to all orders of ϵ [16], and inserting it into the formula above yields $\mu = 1$, independent of D.

Fitting our data according to Eq. (8) yields $\mu = 0.825 \pm 0.1$ (Fig. 11). The accuracy of the fit is similar to the one obtained from the logarithmic *Ansatz* of $E(H)$. Thus, in the three dimensional case one cannot infer the correct expression of the energy barrier from the accuracy of the data fit.

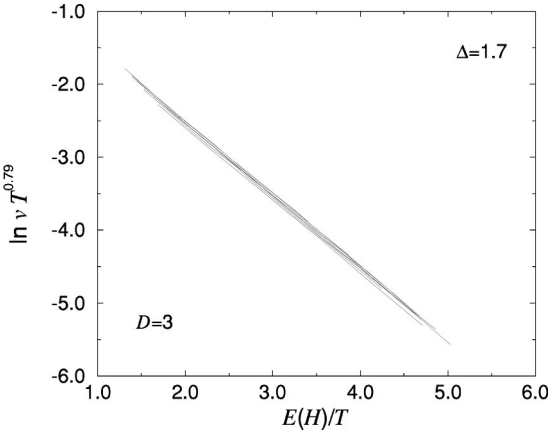

FIG. 9. On rescaling the interface velocities with the numerically determined energy barrier $E(H)$ the data shown in Fig. 7 coalesce onto one single curve. As in the two dimensional case this behavior shows that $c(H) \approx \text{const}$ [see Eq. (5)].

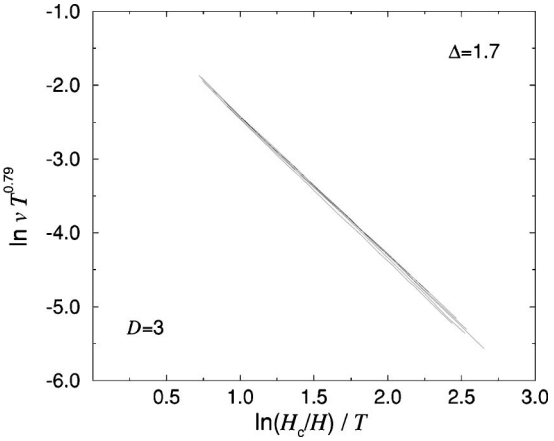

FIG. 10. Equation (6) results in a logarithmic dependence of the energy barrier on the driving field for $\mu \to 0$. The figure shows the interface velocities, which are rescaled according to Eq. (7).

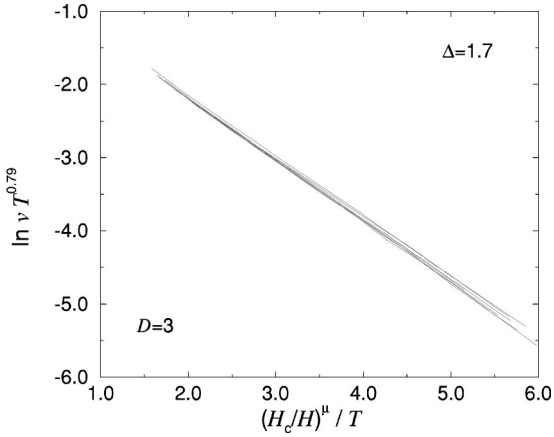

FIG. 11. Scaling plot for the *Ansatz* of the energy barrier according to Eq. (8). From the data collapse one obtains $\mu = 0.825 \pm 0.1$.

On the other hand, the driving fields considered ($H = 0.3, \ldots, 0.6$) are of the same order as the critical value ($H_c \approx 1.37$) while Eq. (8) is believed to be valid only in the limit $H \ll H_c$.

V. DISCUSSION AND CONCLUSION

We investigated numerically the creep motion of a driven interface of a RFIM model in the limit of low temperatures and small driving fields. We found that the interface velocity obeys an Arrhenius law, which was investigated in detail. We assumed that the prefactor of the Arrhenius law can be written as $C(H,T) \sim c(H) T^{-x}$. Applying this *Ansatz* to the numerically determined interface velocities, we find a positive exponent x for the two and three dimensional models. Additionally, our results suggest that $c(H)$ is independent of the driving field in both cases. These results are in contradiction to a renormalization group analysis [11] in which (i) x is claimed to be negative and (ii) $c(H)$ is found to exhibit a significant field dependence [$c(H) \sim H^{\sigma}$ with $\sigma > 0$]. In particular, the opposite sign of the exponent x is remarkable.

Knowing the prefactor $C(H,T)$, it is possible to investigate the driving field dependence of the energy barrier $E(H)$. Our numerical results are in agreement with the assumption that the energy barrier depends logarithmically in both dimensions on the driving field. Again this result is in contradiction with both phenomenological theories and renormalization group approaches, which conjecture an algebraic behavior [11–13]. The logarithmic behavior can can be explained if one assumes that the exponent of the algebraic behavior tends to zero. But in [11–13] a finite value of the corresponding exponent is predicted.

Thus our analyses reveal that the driven interface of a RFIM displays creep motion in the limit of low temperatures and small driving fields characterized by an Arrhenius law as predicted by phenomenological and renormalization group approaches. The details of the Arrhenius law (prefactor and energy barrier) differ, however, from the predicted behavior. Further investigations are needed to understand these differences.

ACKNOWLEDGMENTS

This work was supported by the Deutsche Forschungsgemeinschaft via Graduiertenkolleg ''*Struktur und Dynamik heterogener Systeme*'' at the University of Duisburg, Germany.

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